Slide Set 3

- Carrier drift
- Carrier mobility
- Saturated drift velocity
- Mobility variation with temperature
- Drift current equations
- Band diagrams with an electric field present
- Carrier diffusion
- Flux equation
- Einstein relation
- Total current density
- Carrier recombination and diffusion length

Drift and Diffusion

We now have some idea of the number density of charge carriers (electrons and holes) present in a semiconductor material from the last slide set. Since current is the rate of flow of charge , we shall be able calculate currents flowing in real devices since we know the number of charge carriers. There are two current mechanisms which cause charges to move in semiconductors. These two mechanisms are *drift and diffusion*.

Carrier Drift

- Electron and holes will move under the influence of an applied electric field since the field exert a force on charge carriers (electrons and holes).

$$F = qE$$

- These movements result a current of I_d ;

$$I_d = nqV_dA$$

I_d : drift current

n : number of charge carriers per unit volume

V_d : drift velocity of charge carrier

q : charge of the electron

A : area of the semiconductor

Carrier Mobility , μ

$$V_d = \mu E$$

E : applied field

μ : mobility of charge carrier

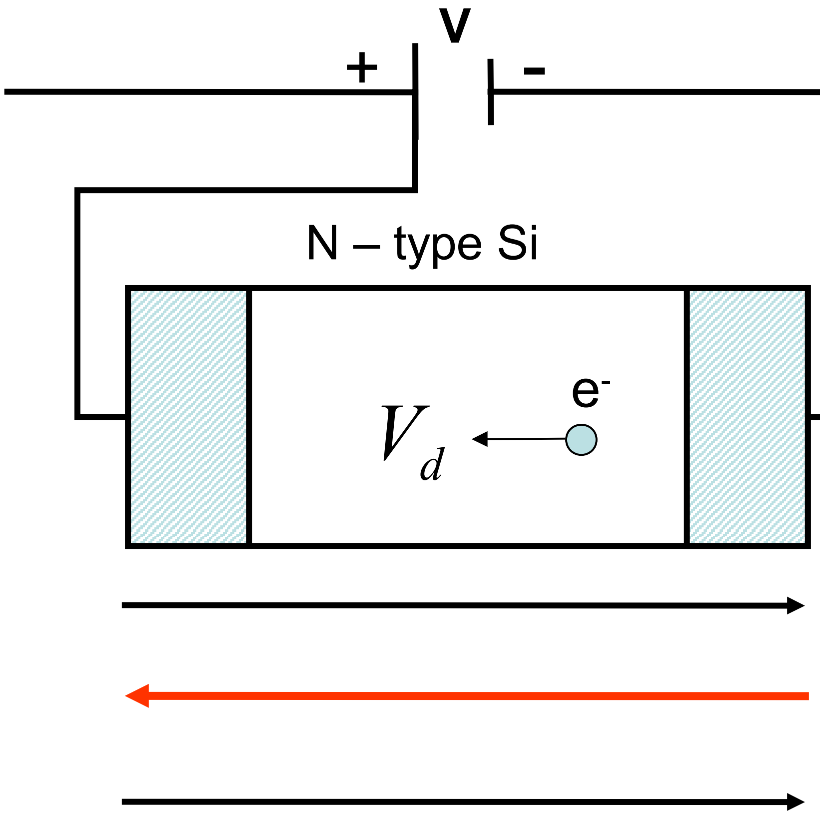
$$\mu = \left[\frac{cm^2}{V \cdot Sec} \right]$$

μ is a proportionality factor

$$\mu = \left[\frac{V_d}{E} \right]$$

- So μ is *a measure how easily charge carriers move* under the influence of an applied field or μ determines *how mobile the charge carriers* are.

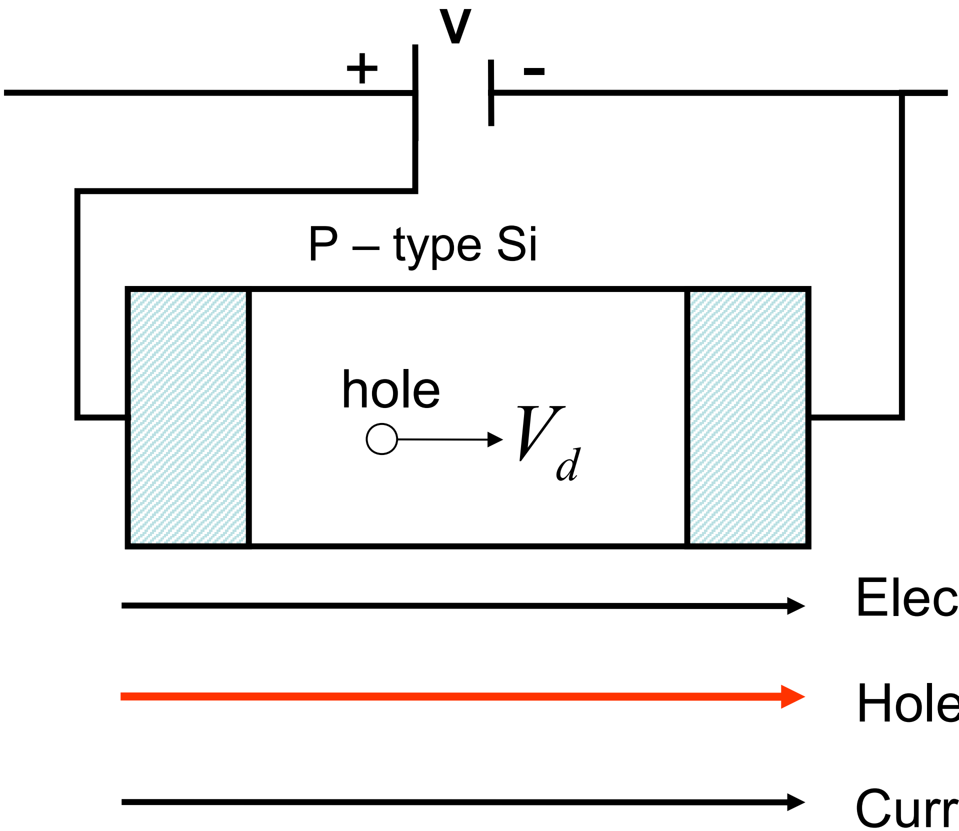
N - type Silicon



$$E = \frac{V}{L}$$

Current carriers are mostly electrons.

P - type Silicon



$$E = \frac{V}{L}$$

Electric field

Hole movement

Current flow

Current carriers are mostly holes.

Carrier Mobility

Macroscopic understanding

$$\mu = \frac{V_d}{E}$$

In a perfect Crystal

$$\rho = 0$$

$$\sigma \rightarrow \infty$$

It is a superconductor

Microscopic understanding? (what the carriers themselves are doing?)

$$\mu = \frac{q\tau}{m^*}$$

$$m_e^* \uparrow m_h^* \quad in\ general$$

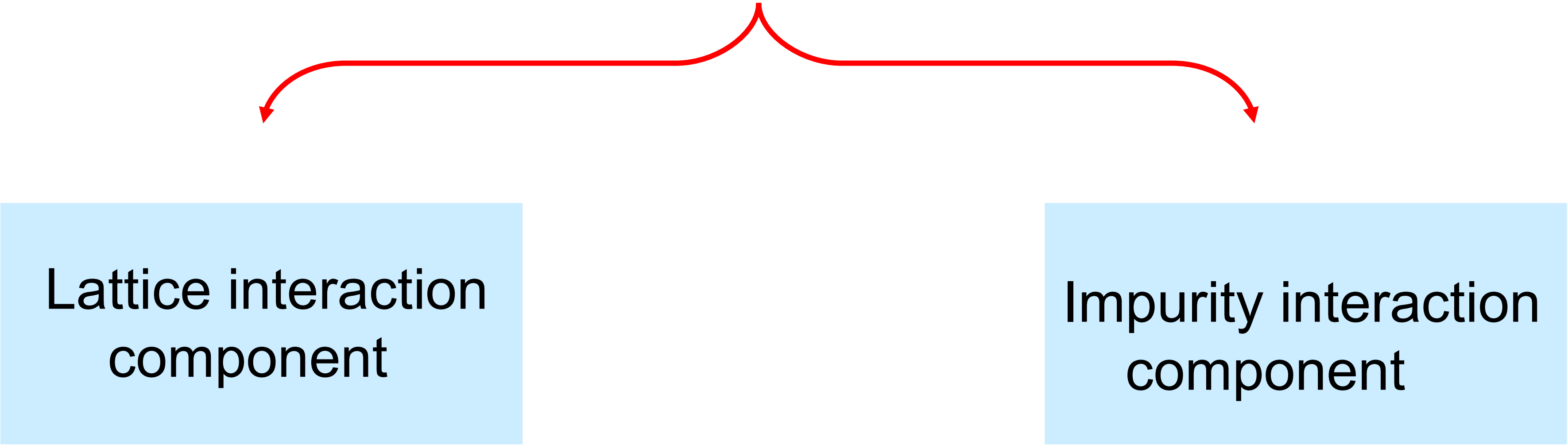
$$m_e^*;\ n - type$$

$$m_h^*;\ p - type$$

- A *perfect crystal has a perfect periodicity* and therefore the potential seen by a carrier in a perfect crystal is completely periodic.
- So the *crystal has no resistance to current flow* and behaves as a *superconductor*. The perfect periodic potential does not *impede the movement of the charge carriers*. However, in a real device or specimen, the presence of impurities, interstitials, subtitionals, temperature , etc. creates a resistance to current flow.
- The presence of all these *upsets the periodicity of the potential* seen by a charge carrier.

Mobility has two components

Mobility has two components



Lattice interaction component

Impurity interaction component

Thermal Velocity

- Assume that s/c crystal is at thermodynamic equilibrium (i.e. there is no applied field). What will be the energy of the electron at a finite temperature?

- The electron will have a thermal energy of k*T*/2 per degree of freedom. So , in 3D, electron will have a thermal energy of

$$E = \frac{3kT}{2} \Rightarrow \frac{1}{2} m^* V_{th}^2 = \frac{3kT}{2} \Rightarrow \boxed{V_{th} = \sqrt{\frac{3kT}{m^*}}}$$

V_{th} : *thermal velocity of electron*

$$V_{th} \propto T^{\frac{1}{2}}$$

$$V_{th} \propto \left(m^*\right)^{-\frac{1}{2}}$$

Random motion result no current.

- Since there is no applied field, the movement of the charge carriers will be completely random. This randomness result **no net current flow**. As a result of thermal energy there are almost an equal number of carriers moving right as left, in as out or up as down.

Calculation

- Calculate the velocity of an electron in a piece of n-type silicon due to its thermal energy at RT and due to the application of an electric field of 1000 V/m across the piece of silicon.

$$V_{th} = ? \qquad RT = 300\ K \qquad m_e^* = 1.18\, m_0$$

$$V_d = ? \qquad E = 1000\ V/m \qquad \mu = 0.15\, m^2/(V-s)$$

$$V_{th} = \sqrt{\frac{3kT\ \text{[redacted]}}{m^*}} \Rightarrow \boxed{V_{th} = 1.08x10^5\, m/\sec}$$

$$V_d = \mu E \Rightarrow \boxed{V_d = 150\, m/\sec}$$

Microscopic understanding of mobility?

❖How long does a carrier move in time before collision ?

The average time taken between collisions is called as relaxation time, ▲ (or mean free time)

❖How far does a carrier move in space (distance) before a collision?

The average distance taken between collisions is called as mean free path, l.

Calculation

Drift velocity=Acceleration x Mean free time

$$V_d = \frac{F}{\text{[redacted]}\,m} \times \tau$$

Force is due to the applied field, F=qE

$$V_d = \frac{F}{\text{[redacted]}\,m} \times \tau = \frac{qE}{\text{[redacted]}\,m}\tau$$

$$V_d = \mu E \Rightarrow \mu = \frac{q\tau}{\text{[redacted]}\,m^*}$$

Calculation

- Calculate the mean free time and mean free path for electrons in a piece of n-type silicon and for holes in a piece of p-type silicon.

$$\tau = ? \qquad l = ? \quad m_e^* = 1.18\, m_o \qquad m_h^* = 0.59 m_o$$

$$\mu_e = 0.15\, m^2/(V-s) \qquad \mu_h = 0.0458\, m^2/(V-s)$$

$$\tau_e = \frac{\mu_e m_e^*}{q} = 10^{-12}\,\mathrm{sec} \qquad \tau_h = \frac{\mu_h m_h^*}{q} = 1.54x10^{-13}\,\mathrm{sec}$$

$$v_{th_{elec}} = 1.08x10^5\, m/s \qquad v_{th_{hole}} = 1.\ 52x10^5\, m/s$$

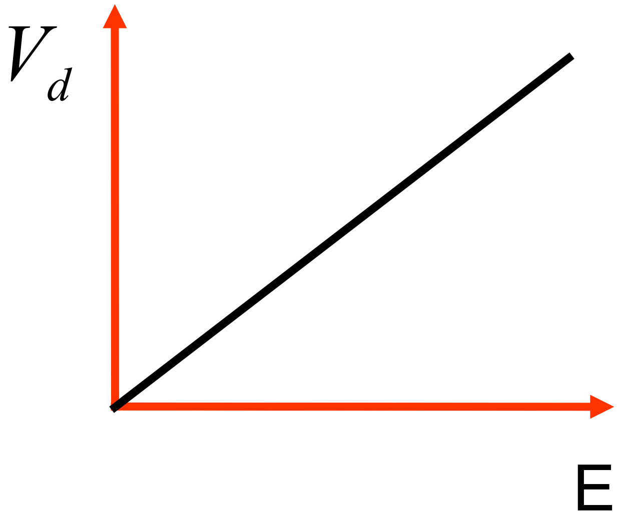
$$l_e = v_{th_{elec}} \tau_e = (1.08x10^5\, m/s)(10^{-12} s) = 10^{-7}\, m$$

$$l_h = v_{th_{hole}} \tau_h = (1.\ 52x10^5\, m/s)(1.54x10^{-13}\,\mathrm{sec}) = 2.34x10^{-8}\, m$$

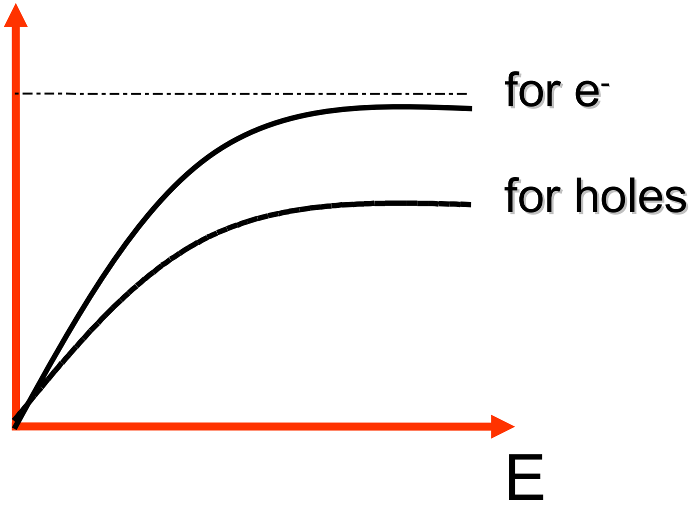
Saturated Drift Velocities

$$V_d = \mu E$$

So one can make a carrier go as fast as we like just by increasing the electric field!!!



(a) Implication of above eqn.



(b) Saturation drift velocity

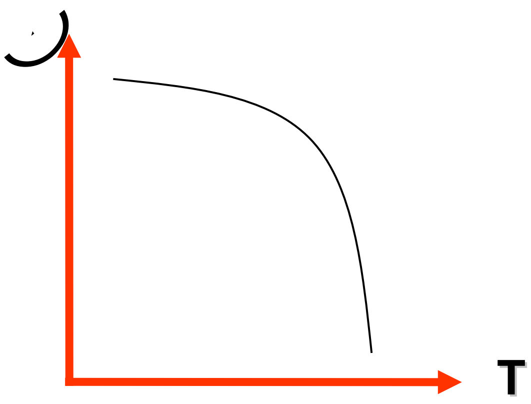
Saturated Drift Velocities

- The equation of $V_d \propto \mu . E$ does not imply that **V_d** increases linearly with applied field **E.**

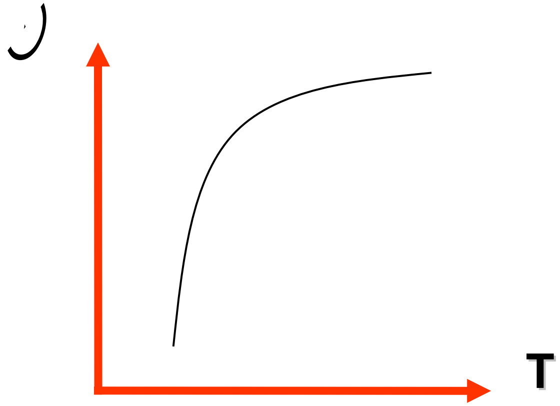
- **V_d** increases linearly for low values of **E** and then it saturates at some value of **V_d** which is close **V_{th}** at higher values of **E**.

- Any further increase in **E** after saturation point does not increase **V_d** instead warms up the crystal.

Mobility variation with Temperature



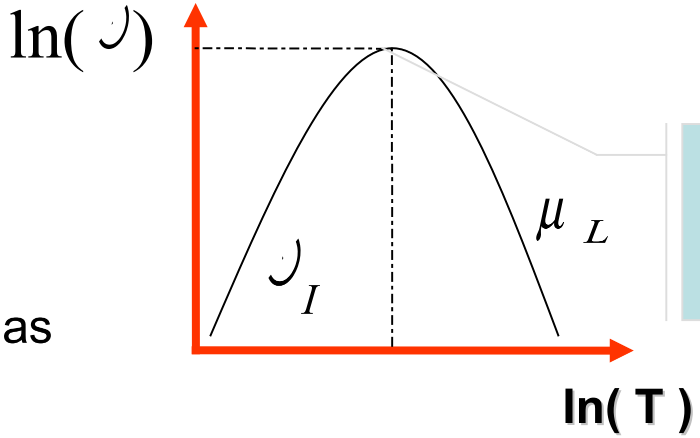
High temperature



Low temperature

$$\frac{1}{\mu_T} = \frac{1}{\mu_L} + \frac{1}{\mu_I}$$

This equation is called as Mattheisen's rule.



Peak depends on the density of impurities

Variation of mobility with temperature

At high temperature
(as the lattice warms up) → μ_L component becomes significant.

μ_L *decreases* when temperature *increases.*

$$\mu_L = C_1 \cdot T^{-\frac{3}{2}} \propto T^{-\frac{3}{2}}$$

C₁ is a constant.

It is called as a $T^{-1.5}$ *power law.*

Carriers are more likely scattered by the lattice atoms.

Variation of mobility with temperature

At low temperatures ⇨ μ_I component is significant.

μ_I *decreases* when temperature *decreases.*

$$\mu_I = C_2 \times T^{\frac{3}{2}}$$

C_2 is a constant.

Carriers are more likely scattered by ionized impurities.

Variation of mobility with temperature

The peak of the mobility curve depends on the number density of ionized impurities.

Highly doped samples will therefore cause more scattering, and have a lower mobility, than low doped samples.

This fact is used in high speed devices called High Electron Mobility Transistors (HEMTs) where electrons are made to move in undoped material, with the resulting high carrier mobilities!

HEMTs are high speed devices.

A Derivation of Ohm's Law

$$I_d = nqV_dA$$

$$V_d = \mu E$$

$$J_d = \frac{\text{[redacted]}}{A}$$

$$\mu = \frac{\text{[redacted]}}{m}$$

$$J_x = nqV_d = nq\mu E$$

$$J_x = \left(\frac{\text{[redacted]}}{m}\right)E_x$$

$$\sigma = \frac{\text{[redacted]}}{m}$$

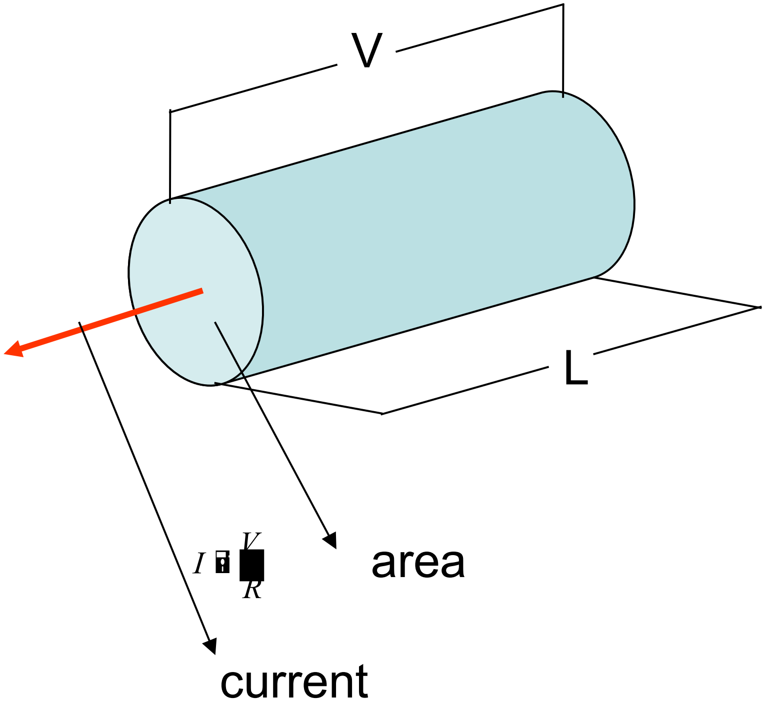
$$J_x = \sigma E_x$$

$$\rho = \frac{\text{[redacted]}}{\sigma}$$

$$[\rho] = [\Omega - m]$$

$$[\sigma] = [\text{[redacted]}(\Omega - m)]$$

A Derivation of Ohm’s Law



This is in fact ohm’s law which is written slightly in a different form.

$$J_x = \sigma E_x \qquad \frac{I_x}{A} = \sigma \frac{V}{L} \Rightarrow \frac{I_x}{A} = \frac{1}{\rho}\frac{V}{L} \qquad I = \frac{VA}{\rho L} = \frac{V}{R}$$

Drift Current Equations

For undoped or intrinsic semiconductor ; $n=p=n_i$

For electron

$$J_n = nqE\mu_n$$

drift current for electrons

number of free electrons per unit volume

mobility of electron

For hole

$$J_p = pqE\mu_p$$

drift current for holes

number of free holes per unit volume

mobility of holes

Drift Current Equations

Total current density

$$J_i = J_e + J_h$$

$$J_i = nqE\mu_n + pqE\mu_P$$

since $n = p = n_i$

$$J_i = n_i q(\mu_n + \mu_p)E$$

For a pure intrinsic semiconductor

Drift Current Equations

J_{total} ▮ ? for doped or extrinsic semiconductor

n-type semiconductor;

$$n >> p \Rightarrow J_T \cong nq\mu_n E = N_D q\mu_n E$$

where N_D is the shallow donor concentration

p-type semiconductor;

$$p >> n \Rightarrow J_T \cong pq\mu_p E = N_A q\mu_p E$$

where N_A is the shallow acceptor concentration

Variation of resistivity with temperature

Why does the *resistivity of a metal increase* with increasing temperature whereas the *resistivity of a semiconductor decrease* with increasing temperature?

$$\rho = \frac{1}{\sigma} = \frac{1}{nq\mu}$$

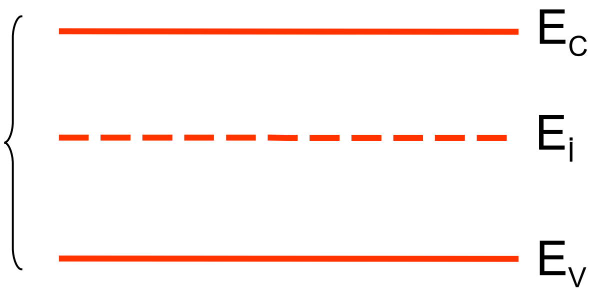
This fact is used in a real semiconductor device called a thermistor, which is used as a temperature sensing element.

The thermistor is a temperature – sensitive resistor; that is its terminal resistance is related to its body temperature. It has a negative temperature coefficient , indicating that its resistance will decrease with an increase in its body temperature.

Band Diagrams with Electric Field Present

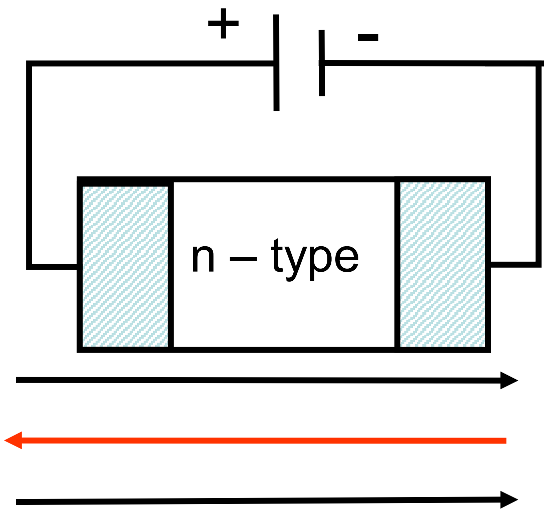
At equilibrium (with no external field)

All these energies are horizontal



Pure/undoped semiconductor

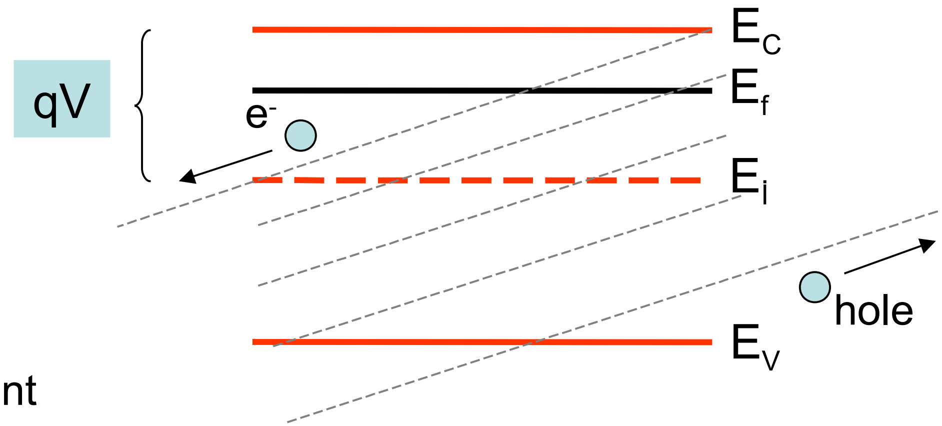
How these energies will change with an applied field ?



Electric field

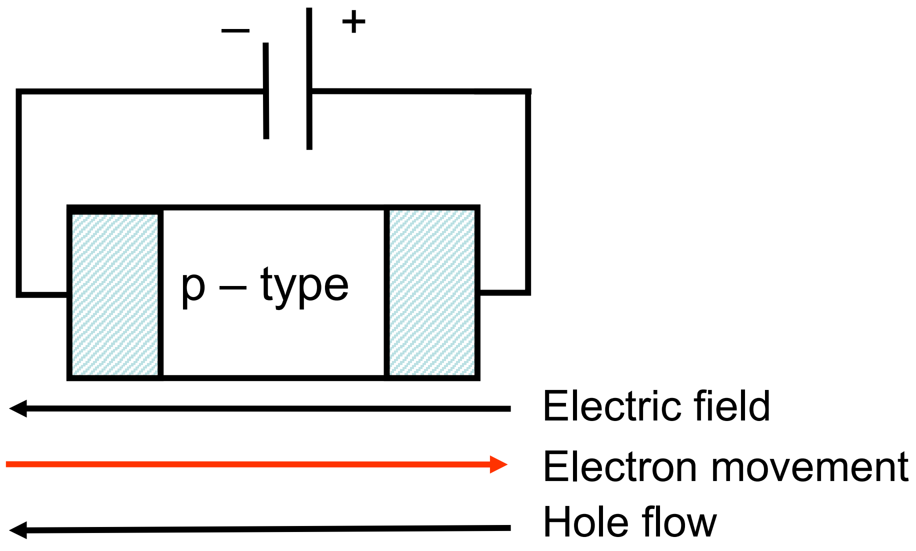
Electron movement

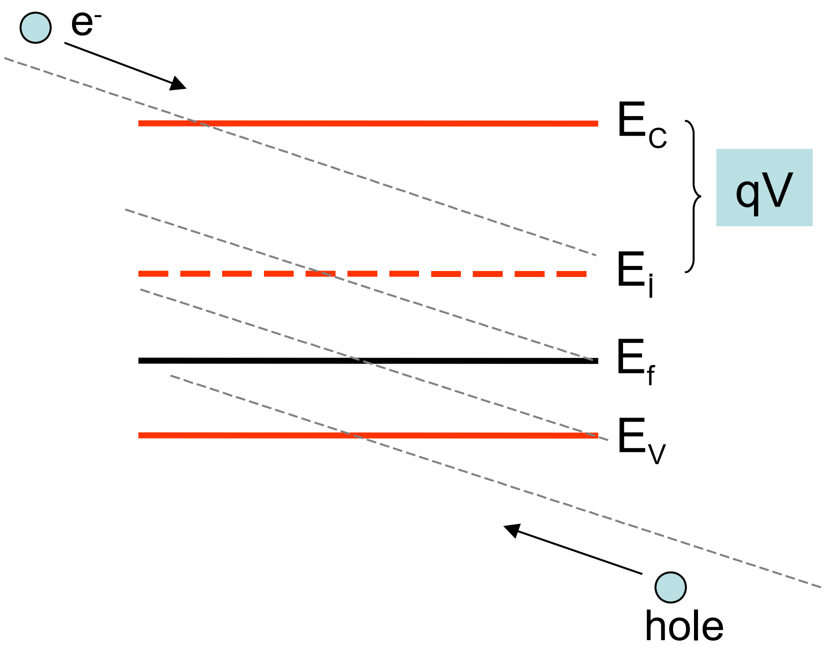
Hole flow



Band Diagrams with Electric Field Present

- With an applied bias the band energies slope down for the given semiconductor. Electrons flow from left to right and holes flow from right to left to have their minimum energies for a p-type semiconductor biased as below.





Under drift conditions

- **Under drift conditions;** holes float and electrons sink. Since there is an applied voltage, currents are flowing and this current is called as drift current.

- There is a certain slope in energy diagrams and the depth of the slope is given by qV, where V is the battery voltage.

Work done on the charge carriers

Work = Force x distance = electrostatic force x distance

$$work = -qE \times L$$

$$= -q\frac{V}{L} \times L \Rightarrow work = -qV = \boxed{gain\ in\ energy}$$

$$Slope\ of\ the\ band = -\frac{qV}{L} = -qE = \boxed{Force\ on\ the\ electron}$$

$$\boxed{E = \frac{V}{L}}$$

where L is the length of the s/c.

Since there is a certain slope in the energies, i.e. the energies are not horizontal, the currents are able to flow.

Work done on the charge carriers

Electrostatic Force = − *gradiant of potential energy* = − [redacted]

$-qE_x = -$ [redacted] $\Rightarrow E_x =$ [redacted] $\cdot$ [redacted] (1)

one can define electron's electrostatic potential as

$E_x = -$ [redacted] (2)

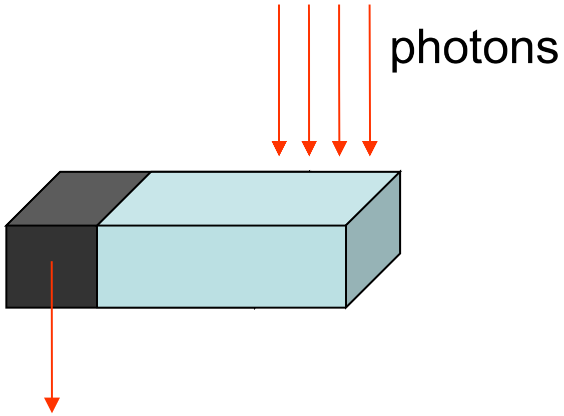
comparison of equations (1) *and* (2) *gives,*

$V_n = -$ [redacted] *is a relation between* V_n *and* E_i

Carrier Diffusion

Current mechanisms

- Drift
- Diffusion



Contact with a metal

$$P = nkT$$

$$\frac{dP}{dx} = \frac{dn}{dx} kT$$

$$\frac{dn}{dx} = \frac{1}{kT} \frac{dP}{dx}$$

Carrier Diffusion

- Diffusion current is due to the movement of the carriers from high concentration region towards to low concentration region. As the carriers diffuse, a diffusion current flows. The force behind the diffusion current is the ***random thermal motion of carriers***.

$$\frac{dn}{dx} = \frac{1}{kT} \cdot \frac{dP}{dx}$$

- A concentration gradient produces a pressure gradient which produces the force on the charge carriers causing to move them.

How can we produce a concentration gradient in a semiconductor?

1) By making a semiconductor or metal contact.
2) By illuminating a portion of the semiconductor with light.

Illuminating a portion of the semiconductor with light

- By means of illumination, electron-hole pairs can be produced when the photon energy > E_g.

- So the increased number of electron-hole pairs move towards to the lower concentration region until they reach to their equilibrium values. So there is a number of charge carriers crossing per unit area per unit time, which is called as flux. Flux is proportional to the concentration gradient, dn/dx.

$$Flux = -D_n \frac{dn}{dx}$$

Flux

$$[Flux] = m^{-2} - s^{-1}$$

$D = v_{th} l, \ [D] = m^{2}$ [redacted] s

The current densities for electrons and holes

$J_n = -q\Big(-D_n \, dn$ [redacted] $dx \Big) = qD_n \, dn$ [redacted] dx *for electrons*

$J_p = +q\Big(-D_p \, dp$ [redacted] $dx \Big) = -qD_p \, dp$ [redacted] dx *for holes*

$[J_{n,p}] = [$ [redacted] $m^2]$

Einstein Relation

Einstein relation relates the two independent current mechanicms of mobility with diffusion;

$$\frac{D_n}{\mu_n} = \boxed{\frac{kT}{q}} \quad \textit{and} \quad \frac{D_p}{\mu_p} = \boxed{\frac{kT}{q}} \quad \textit{for electrons and holes}$$

Constant value at a fixed temperature

$$\frac{\cancel{cm^2}/\sec}{\cancel{cm^2}/V-\sec} = volt \qquad \frac{kT}{q} = \frac{(J/K)(K)}{C} = volt$$

$$\frac{kT}{q} = 25\ mV \quad \textit{at room temperature}$$

Total Current Density

When both electric field (gradient of electric potential) and concentration gradient present, the total current density ;

$$J_n = q\mu_n nE + qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p pE - qD_p \frac{dp}{dx}$$

$$J_{total} = J_n + J_p$$

Carrier recombination and diffusion length

- By means of introducing excess carriers into an intrinsic s/c, the number of majority carriers hardly changes, but the number of minority carriers increases from a low- to high-value.

- When we illuminate our sample (n-type silicon with 10^{15} cm^{-3}) with light that produces 10^{14} cm^{-3} electron-hole pairs.

- The electron concentration (majority carriers) hardly changes, however hole concentration (minority carriers) goes from 1.96×10^{5} to 10^{14} cm^{-3}.

Recombination Rate

- Minority carriers find themselves surrounded by very high concentration of majority carriers and will readily recombine with them.
- The recombination rate is proportional to excess carrier density, δ_p .

$$\frac{d\delta p}{dt} = -\frac{1}{\tau_p}\delta p \Rightarrow \delta p(t) = \delta p(0)\exp\left(-\frac{t}{\tau}\right)$$

Lifetime of holes

Excess hole concentration when t=0

Excess hole concentration decay exponentially with time.

Similarly, for electrons;

$$\frac{d\delta n}{dt} = -\frac{1}{\tau_n}\delta n \Rightarrow \delta n(t) = \delta n(0)\exp\left(-\frac{t}{\tau}\right)$$

Diffusion Length L

When excess carriers are generated in a specimen, the minority carriers diffuse a distance, a characteristic length, over which minority carriers can diffuse before recombining majority carriers. This is called as a diffusion length, L.

Excess minority carriers decay exponentially with diffusion distance.

$$\delta n(x) = \delta n(0)\exp\left(-\frac{x}{L_n}\right)$$

Excess electron concentration when x=0

$$\delta p(x) = \delta p(0)\exp\left(-\frac{x}{L_p}\right)$$

Diffusion length for electrons

$$L_n = \sqrt{D_n \tau_n}$$

Diffusion length for holes

$$L_p = \sqrt{D_p \tau_p}$$